

Name: \_\_\_\_\_

**Math 235 - Calculus III - Spring 2012**

**Exam 1**

INSTRUCTIONS: Show all your work to receive full credit. Books and notes are not allowed.

1. (10 points) Find all the  $t$ -values where the parametric curve has a horizontal tangent and all the  $t$ -values where the parametric curve has a vertical tangent.

$$x(t) = 2 \sin t \quad y(t) = 4 \cos(t)$$

2. (5 points) Which of the following is the equation of  $r = \frac{6}{3 \cos \theta + 2 \sin \theta}$  expressed in rectangular coordinates

(a)  $y = \frac{6}{3x+2y}$

(b)  $\sqrt{x^2 + y^2} = \frac{6}{3x+2y}$

(c)  $y = 3 - 1.5x$

(d)  $y^2 = \frac{36}{3x+2y} - x^2$

3. (10 points) Find the area inside the cardioid  $r = 1 + \cos \theta$  and outside the circle  $r = \cos \theta$

4. (7 points) Write down *but do not integrate* an expression for the arc length of the polar curve  $r = \sqrt{\theta}$  over the interval from 0 to  $\pi/2$ .

5. (10 points) Find the equation of the ellipse that has foci at  $(-1, 1)$  and  $(-1, 3)$  and minor axis of length 4.

6. (8 points each) Given the vector  $\mathbf{v} = \langle 2, 6, 2 \rangle$  and the vector  $\mathbf{u} = \langle 1, 1, 2 \rangle$ , compute the following

(a)  $\mathbf{u} \cdot \mathbf{v}$ . What does this tell you about the angle between  $\mathbf{u}$  and  $\mathbf{v}$ ?

(b)  $\mathbf{u} \times \mathbf{v}$

(c)  $\text{proj}_{\mathbf{u}} \mathbf{v}$

7. (10 points) Find the point of intersection of the line and plane.

$$\begin{aligned}x &= 1 + t & y &= -1 + 3t & z &= 2 + 4t \\x - y + 4z &= 7\end{aligned}$$

8. (10 points) Find the equation of the plane that contains the point  $(2, 0, 3)$  and the line  $x = -1 + t$   $y = t$   $z = -4 + 2t$

9. (10 points) Draw a picture that illustrates where the conversions from spherical coordinates to cylindrical coordinates come from.