Math 236-Differential Equations
Take home exam 1

Instructions: You are expected to work alone on this exam. You may consult your notes, textbook, or me (Maria) for help. Your final product should be **word processed** and should include **Berkeley Madonna** output. You should also attach a piece of paper with the following statement and your signature:

"I certify that I have completed this exam without consulting any person other than my partner and the instructor of this course or any materials besides the notes and textbook.".

Problem 1 - Heating and Cooling a Building

Consider the problem of heating or cooling a building. The DE that governs the temperature inside the building is a modified version of Newton's Law of Cooling given by

$$\frac{dT}{dt} = k[M - T] + H(t) + k_u[T_D - T]$$

The M is the environmental temperature, $H(t)$ is a function is a function that describes temperature changes that have to do with things inside the building (people, computers, etc), T_D

is the desired temperature of the building (where the thermostat is set), and k_u is a measure of the efficiency of the HVAC system. We call $1/k$ the time constant for the building without heating and cooling. The time constant for the building with heating and cooling is $1/k_1$, where $k_1 = k + k_u$. .

1. For now, let's ignore the $H(t)$ term. Choose values for the parameters M, k, k_u, and T_D and for $T(0)$.Typically the value of $1/k$ is 2-4 hours and typically values for k_u are around 2. Run the model in Berkeley Madonna and explain your output. In particular, does the building ever reach the desired temperature? How long does it take for the building to reach a steady temperature?
   (a) Investigate what happens to the curves when you change each of the parameters individually. In particular, how are final temperature and the time to reach final temperature related to changes in each parameter? (You may want to do a batch run in BM to get this)
   (b) Now using a paper and pencil and leaving all parameters unknown, find an expression for the final temperature and the time to reach 0.1 degree above or below final temperature in terms of M, k, k_u, T_D, and T_0. (Even your response to this question should be done on a word processor)
   (c) Let's see what happens when the outside temperature is no longer a constant. A typical way to represent daily temperature is a sine curve with period 24. Suppose we want to model an exterior temperature similar to early March. We can assume that the high temperature is around 60 while the low is around 40 and that the high and low temperatures occur 12 hours apart. Fit a sine or cosine function to this exterior temperature and then replace M with that function. What happens to the model now? Try different values for k and k_u. What happens? If you are an engineer designing a building, how do you think you want to choose k and k_u?
   (d) Finally, let's add an $H(t)$. Use the model in part (c) and add an $H(t)$ term that you think represents the temperature changes due to objects and people inside the building. How does that change your output? (There is no single right answer for $H(t)$, but you should give reasons for your choice)

Problem 2 - Mixing Problem

Consider a reservoir of 8 billion gallons with inflow and outflow rates of 500 million gallons per day. There is a possibility of a pollutant entering the reservoir with the inflow, but that is not constant.

2. Suppose that the concentration of pollutant flowing in can be modelled by a decaying exponential function $p(t) = p_0 e^{-kt}$. Choose a value for the initial concentration of pollutant in the reservoir and a value for k and run the model. What does the solution look like? How long does it take for the pollutant to be flushed out of the system? Try various values of k, how does varying k change your output? Relate this to what happens in the reservoir.

3. If you know the initial concentration in the reservoir is $x(0) = x_0$ and you know the concentration at some time t_0 is $x(t_0)$, can you determine p_0 if k is known? (this takes calculation, not running on BM). What would solving for p_0 mean in the real world?